

HYDRODYNAMICS OF SOIL WATER-THEORETICAL APPROACH

HIDRODINAMICA APEI IN SOL-ABORDARE TEORETICA

MATEI FLORICA, IOANA POP, LIANA STANA, MARIA MICULA

USAMV Cluj-Napoca, Faculty of Horticulture,
Dept. of Mathematics and Computer Science

Abstract. *The variable amount of water contained in a unit of mass or volume of soil is an important factor affecting the growth of plants. In this paper we will study the hydrodynamics of soil water considering the macroscopic and saturated characterization of the soil.*

Rezumat. *Cantitatea de apa continuta in sol este un factor important care afecteaza cresterea plantelor. In aceasta lucrare ne propunem sa studiem hidrodinamica apei in sol folosind caracterizarea macroscopica a solului.*

Soil is an important reservoir of fresh water. Soil transforms noncontinuous rainfall or snow into a continuous flow of water to the roots of the plants. The retention capacity of soil able to sequester rain water is approximatively equal to the capacity of all lakes. Transport of water and soluble materials occurring naturally or anthropogenically and is linked to hydrologic process. Water is important for plant function; constituents within soil are mobilized and transported as a result of precipitation or irrigation. Other role played by water is to transport agents for chemicals pollution. The distance and rate of their motion on and bellow the soil surface depends of hydrological events. The quality of continental water resources in space and time is greatly influenced by soil hydrology.

For all process described above is very important to have mathematical models that can describe these processes [4]. For the mathematical model is necessary to find some reasonable assumption about the soil system and water flow.

If the system is in equilibrium no flow will occur. If we do not have equilibrium the flow will occur from region of high to low hydraulic head. If we assume that soil system is at macroscopic scale and saturated the primary flow equation is the Darcy's equation. When Darcy's equation is combined with conservation mass the result is the continuity equation. The continuity equation can have several different form; these forms are generically known as soil water flow equations [5].

If we assume that soil is an unsaturated system we use the Richards equation.

In this note we work with soil system at macroscopic scale and saturated conditions.

BERNOULLI AND POISEUILLE EQUATIONS

In order to understand Darcy's equation we start with classical relationships from fluid dynamics the Bernoulli and Poiseuille laws.

Bernoulli equation relates the total potential for ideal fluids (non-viscous fluid), which is one that is incompressible and which exhibits steady and irrotational flow.

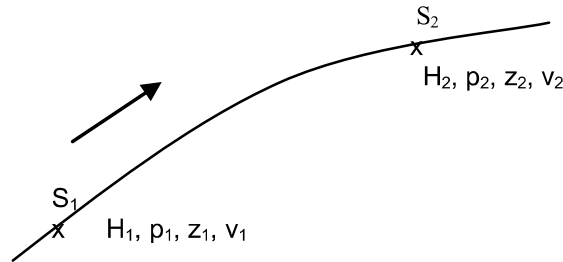


Figure 1 Position and head along the streamline

If p , z , v represent pressure elevation and velocity at points S_1 and S_2 , the sum of gravitational, pressure and inertial energy at positions S_1 and S_2 are the same along any streamline:

$$\frac{p_1}{\rho g} + z_1 + \frac{v_1^2}{2g} = \frac{p_2}{\rho g} + z_2 + \frac{v_2^2}{2g} \quad (\text{ideal fluids}) \quad (1)$$

For real fluids viscosity produce a loss of energy that must be overcome. For most problems of interests in soil the velocity head will be negligible compared with pressure and gravitational terms, and the Bernoulli equation is replaced by

$$\frac{p_1}{\rho g} + z_1 + \Delta H = \frac{p_2}{\rho g} + z_2 \quad (\text{real fluids}) \quad (2)$$

where $\Delta H = H_2 - H_1$, H_1 , H_2 are hydraulic heads. If the flow is from S_1 to S_2 then H_1 is greater than H_2 and ΔH become negative. Relation (2) will be used for porous media flow.

Poiseuille's laws relates flow rate to the loss of energy in a horizontal tube.

$$Q = -\frac{\pi R^4 \Delta p}{8\eta \Delta x} \quad (3)$$

where

Q represents discharge rate (m^3/s),

R represents radius of tube,

η represents viscosity ($kgm^{-1}s^{-1}$),

Δp represents pressure gradient along tube.

Negative sign means that fluid flow from higher to lower pressure and Q is positive for flow in the horizontal x direction.

Flux density represents flow per unit area and is denoted by J_w . Relation (3) becomes

$$J_w = -K^* \frac{\Delta h}{\Delta x} \quad (4)$$

where $K^* = \frac{\rho g R^2}{8\eta}$, thus J_w is proportional to the pressure head gradient $\frac{\Delta h}{\Delta x}$ and J_w is inversely proportional to the viscosity.

DARCY'S EQUATION

In 1859 Darcy experimentally demonstrated for columns of sand a linear relationship between the flux density J_w and hydraulic gradient, the Darcy's equation for saturated flow is

$$J_w = -K_s \nabla H \quad (5)$$

Equation (5) shows that the flux density is proportional to the driving force of the water flow which is the gradient of the potential. In the literature J_w is also called Darcian velocity [$L \cdot T^{-1}$]. The negative sign in equation (5) means that the water flows in the direction of the decreasing potential. The constant value K_s depends upon the nature of the soil and is numerically equal with flow rate when the hydraulic gradient is unity.

The values of K_s are between $0.1 \text{ cm} \cdot \text{day}^{-1} (10^{-8} \text{ ms}^{-1})$ and $10^2 \text{ cm} \cdot \text{day}^{-1} (10^{-5} \text{ ms}^{-1})$.

Poiseuille equation (4) and Darcy equation (5) seem to be the same but they are not. Darcy equation is an empirical equations and it is expressed in volume of flow per unit area per unit time. Other assumption used is that of isotropic medium, otherwise the hydraulic conductivity value depends on the flow direction.

It is important to know when we can use the Darcy equation. For a high or low gradient the Darcy equation does not work. For high velocities J_w becomes nonlinear respects with hydraulic gradient because of turbulent flow.

A mathematical criterion to distinguish laminar flow by turbulent flow is given by Reynolds numbers:

$$R_e = \frac{|J_w| \rho d}{\eta} \quad (6)$$

Where d is the effective pore diameter. For $R_e < 1$ laminar condition are expected. In all soils other than sand d is not completely determined so is very difficult to determine the Reynolds number and there are used other criterions.

CONCLUSIONS

In this section we discuss some aspects regarding the applicability of Darcy equation.

If we study the soil system at microscopic scale the flow in each individual pore is considered and for each defined continuous pores the Navier – Stokes equations apply. For the solution we do not have detailed description of the geometrical characterization of individual pores to obtain a solution for the representative

elementary volume. Even if we find these geometrical details the voluminous calculations will be necessary even for a small representative elementary volume.

The macroscopic approach of water transport relates with the entire cross section of the soil with the condition of representative elementary volume is satisfactory. Water does not flow through entire macroscopic area, it flows only in the area not occupied by the soil phase and by the air phase; in that case we deal with unsaturated soils.

We have assumed that we have a saturated with water, inert rigid soil. Water is flowing through all pores of the soil under a positive pressure head. In fields this situation is rarely find. Usually it is quasi – saturated with the soil water content $\theta_w = mP$ where $m \in [0.85, 0.95]$ at $H \geq 0$ and P is the porosity. The air occupies $P(1 - m)$ volume and it is not considered.

Laminar flow prevails only at relatively low flow velocities and narrow tube. Conveniently laminar flow is the rule rather than the exception in most water flow process taking place in soil because of the narrowness of soil pores [1].

In order to solve equation (5) we have to measure saturated hydraulic conductivity, that is one of the principal soil characteristics and for it determination only direct measurements are appropriate [2].

The Darcy's equation may be extended to the layered soils system for cases when the flow is parallel, perpendicular or is an angle less than 90° with the layer.

In the scientific literature deviations from the Darcy equation was observed within pure clay having very large specific surfaces (e. g. $10^2 \text{ m}^2 \cdot \text{g}^{-1}$). The reasons for this behavior are:

- Clay particles shift and the clay paste consolidate due to the imposed hydraulic gradient and the flow of water.
- Viscosity of water close to the clay surface is different than that of bulk water or that in the center of the larger soil pores.
- The coupling of the transfer of water, heat and solute.

Other case when the Darcy's equation does not work was discussed in [3] for losses.

With all this difficulties emphasized in this note Darcy's equation is either exact or at least a very good approximation for soil water hydrodynamics.

REFERENCES

1. Hillel D., 2003 - *Introduction to environmental soil physics*. Springer Verlag
2. Miroslav Kutilek, Donald R. Nielsen, 1994 - *Soil Hydrology*. Catena Verlag.
3. Matei Florica, Pop Ioana, Maria Micula, V. Budiu, M. Dirja, 2007 - *Parameter determination of a mathematical model for soil moisture*. Bull. USAMV-CN, Horticulture, vol 64(1-2) pp575-580.
4. Thornley J. H. M., J France, 2004 - *Mathematical models in agriculture*. Cabi Press.
5. Warrick A. W., 2003 - *Soil water dynamics*. Oxford Univ. Press.